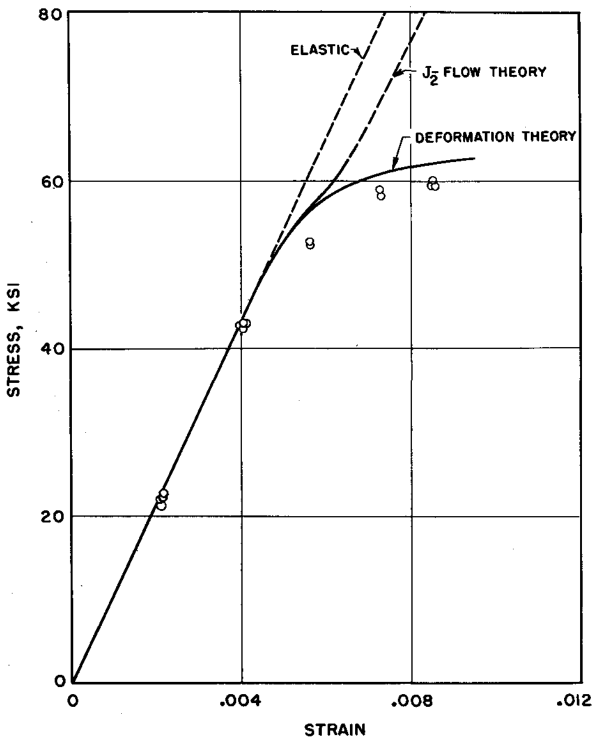

Fig. 3 The test results of experiments in plastic buckling using plastic strains of substantial magnitudes.

produces an accentuating influence in the development of the anisotropic yield surface. In fact we believe that for a smaller initial imperfection an anisotropic yield surface will tend to produce the same results as a large initial imperfection will tend to give with the J_2-theory, which may also explain some questions raised in the literature.[7-9] It is not important whether the initial yield surface has or has not a corner at the σ-axis. It is important how fast the slope $d\gamma''/d\varepsilon''$ changes with the development of the shearing plastic strains. This change is extremely fast, much more than is expected when the J_2-theory is applied.

Additional experiments with present day instrumentation as well as theoretical work is necessary to finally verify these thoughts. For example, experiments on plastic buckling could be performed with the same material used for obtaining the yield surfaces, and then theoretical calculations could be made with the exactly obtained results. Such experiments and theoretical work are now in progress.

3. Final Conclusions

It is seen that a very small plastic strain of the order of a few hundred μin. per inch produces a severe change in the yield surface, that the material becomes anisotropic very fast and the yield surface should be the one of anisotropic flow theory where the anisotropy is due to the motion and deformaton of the yield surface. We believe also that when real yield surfaces are introduced into the picture the problem of plastic buckling may be better understood. Indeed, as Hutchinson and Koiter[10] point out in a recent survey "it is somewhat doubtful that the simplest J_2-theory would be adequate for buckling problems." Additional proper experimental and appropriate theoretical work is necessary.

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Equation for Nonlinear Vibrations of Shells

N. ANDERSON*

University of York, Heslington, York, England

THE nonlinear differential equation

$$\ddot{x}+\varepsilon x(x\ddot{x}+\dot{x}^2)+x=0, \qquad \varepsilon>0, \quad 0\leqq t<\infty \tag{1}$$

arises[1] in the theory of nonlinear inextensional vibrations of an infinitely long cylindrical shell. The purpose of this Note is to study the phase plane behavior of the solutions of Eq. (1) and in particular to obtain the period of the motion.

If we write

$$\dot{x}=U \tag{2}$$

then we can write Eq. (1) in the form

$$\dot{U}=-x[(1+\varepsilon U^2)/(1+\varepsilon x^2)] \tag{3}$$

The differential equation for the solution in the (x, U) phase plane is

$$dU/dx=\dot{U}/\dot{x}=-(x/U)[(1+\varepsilon U^2)/(1+\varepsilon x^2)] \tag{4}$$

which can be integrated to give the one parameter family of phase-paths

$$(1+\varepsilon x^2)(1+\varepsilon U^2)=C \tag{5}$$

where C is a constant of integration to be determined from the initial conditions of the problem by

$$C=\{1+\varepsilon[x(0)]^2\}\{1+\varepsilon[\dot{x}(0)]^2\} \tag{6}$$

These phase-paths are a family of closed curves which are symmetric with respect to x and U and represent periodic motions.

From Eq. (4) we see that the origin of the phase plane, which

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* Lecturer in Mathematics, Department of Mathematics.

itself is a phase path representing the solution $x \equiv 0$, is a singular point. In the neighborhood of the origin the phase-paths have the form

$$U^2 + x^2 = D = \text{const} \tag{7}$$

of a family of circles. By comparison of Eq. (7) with the simple harmonic motion phase-paths

$$U^2 + \omega^2 x^2 = \text{const} \tag{8}$$

we see that for values of C close to 1 the motion is simple harmonic with period 2π.

If we write the phase-paths Eq. (5) in polar coordinates (r, θ) we obtain

$$r^2(1 + \varepsilon r^2 \sin^2 2\theta/4) = (C-1)/\varepsilon \tag{9}$$

and we see that the curves have a maximum radius $r_{\max} = [(C-1)/\varepsilon]^{1/2}$ at $\theta = 0$, $\pm\pi/2$, π and a minimum radius $r_{\min} = [2(C^{1/2}-1)/\varepsilon]^{1/2}$ at $\theta = \pm\pi/4$, $\pm 3\pi/4$.

The period of a particular oscillation can be written in the form

$$T = 4\int_0^{r_{\max}} \frac{dx}{U} = 4\varepsilon^{1/2} \int_0^{r_{\max}} \frac{dx}{[C/(1+\varepsilon x^2) - 1]^{1/2}} \tag{10}$$

If we make the successive substitutions

$$z = C/(1+\varepsilon x^2) - 1$$

$$\phi = \tan^{-1}(z^{1/2})$$

Eq. (10) reduces to

$$T = \frac{4}{\alpha}\int_0^{\phi_1} \frac{\cos^2\phi \, d\phi}{(1-p^2\sin^2\phi)^{1/2}} \tag{11}$$

where

$$\alpha^2 = (C-1)/C^2, \quad p^2 = C/(C-1), \quad \phi_1 = \tan^{-1}[(C-1)^{1/2}] \tag{12}$$

If we now make the substitution $p \sin\phi = \sin\mu$

$$T = 4C^{1/2}\int_0^{\pi/2} (1 - p^{-2}\sin^2\mu)^{1/2} d\mu = 4C^{1/2}E(p^{-1}) \tag{13}$$

where E is a complete elliptic integral of the second kind.

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Measurement of the Error of Temperature Sensors in Flowing Gases

C. R. Rein*
Naval Coastal Systems Laboratory, Panama City, Fla.
AND
J. R. O'Loughlin†
Tulane University, New Orleans, La.

Nomenclature

D = wire diameter
h = convective coefficient
k = thermal conductivity of wire
Nu = Nusselt number
Pr = Prandtl number
Re = Reynolds number
T = absolute temperature
T_s = effective black body temperature of the environment of the sensor as seen by the sensor
V = gas velocity
x = coordinate along wire axis
α = absorptivity of sensor
ε = emissivity of sensor
σ = Stefan-Boltzmann constant

Subscripts

1, 2 = smaller diameter and larger diameter wire, respectively
g = any gas
a = air

When a temperature transducer is placed in a flowing gas and the gas is not in thermal equilibrium with its environment, the temperature transducer does not, in general, achieve the gas temperature. It is then desirable to relate the temperature indicated by the transducer to the actual gas temperature. Often a calculation of the transducer error is impossible because radiative environment properties are unknown.

Consider a butt-welded wire thermocouple along an isotherm in a flowing gas. It is assumed that the thermocouple junction is in a region of the wire approximately free of temperature gradients or variations of gas flow and properties. The total emissivities and absorbtivities of the thermocouple materials are assumed to be equal and constant. Catalytic reactions are assumed negligible. The wire is assumed to be a circular cylinder. The axial coordinate, x, coincides with the centerline of the wire.

The energy balance equation for the wire near the thermocouple junction is

$$(DK/4)(\partial^2 T/\partial x^2) + h(T_g - T) - \varepsilon\sigma T^4 + \alpha\sigma T_s^4 = 0 \tag{1}$$

The first term of Eq. (1) can be set equal to zero near the thermocouple junction because it has been assumed that the temperature does not vary with x. Then differentiating the remaining terms with respect to wire diameter at constant gas temperature and velocity, and solving for gas temperature yields

$$T_g = T + (h + 4\varepsilon\sigma T^3)[(\partial T/\partial D)_{T_g,V}]/[(\partial h/\partial D)_{T_g,V}] \tag{2}$$

The right side of Eq. (2) can be regarded as the thermocouple temperature measurement, T, plus the correction term. Significantly, Eq. (2) does not explicitly contain T_s, a potentially complicated function of the radiative environment, because the fourth term of Eq. (1) is independent of wire diameter. This fundamental advantage of Eq. (2) over Eq. (1) in evaluating T_g applies whether or not the radiative environment is uniform on all sides. However, the two partial derivatives in the correction term of Eq. (2) must be evaluated, in addition to the convective coefficient and the emissivity, in order to evaluate the correction term. Means of evaluating the partial derivatives will now be proposed.

The partial derivative $[(\partial T/\partial D_{T_g,V})]$ can be approximated in the following way, regardless of the kind of gas, when two thermocouples with different diameters are used.

$$[(\partial T/\partial D)_{T_g,V}] \approx (T_2 - T_1)/(D_2 - D_1) \tag{3}$$

Temperature measurements with the thermocouples yield T_1 and T_2. The measurement of D_1 and D_2 can be accomplished with a micrometer. The partial derivative $[(\partial h/\partial D)_{T_g,V}]$ can be evaluated with the aid of the relation among the Nusselt, Prandtl, and Reynolds numbers which is valid for the particular gas and sensor shape.

As an example, for circular cylindrical wires in air at Reynolds numbers between 1 and 1000, the McAdams relation[1] is available

$$(Nu)(Pr)^{-0.3} = 0.35 + 0.56(Re)^{0.52} \tag{4}$$

The expression for $[(\partial h/\partial D)_{T_g,V}]$ can be derived directly from Eq. (4) but a simplification is often possible. Suppose

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* Research Mechanical Engineer; formerly Graduate Student, Mechanical Engineering Department, Tulane University.
† Associate Professor, Mechanical Engineering Department; presently at Purdue University, Indianapolis, Indiana.